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Boundary Fractal Analysis of Two Cube-oriented Grains in Partly Recrystallized Copper

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Abstract. The protrusions and retrusions observed on the recrystallizing boundaries affect the migration kinetics during recrystallization. Characterization of the boundary roughness is necessary in order to evaluate their effects. This roughness has a structure that can be characterized by fractal analysis, and in this study the so-called “Minkowski sausage” method is adopted. Hereby, two cube-oriented grains in partly recrystallized microstructures are analyzed and quantitative information regarding the dimensions of protrusions/retrusions is obtained.

1. Introduction

During recrystallization of deformed metals, new almost perfect nuclei form in the deformed matrix and grow by boundary migration until the entire deformed matrix is replaced by recrystallized grains. Traditionally it is considered that the nuclei form and grow in a relatively homogeneous manner. Modern techniques including synchrotron X-rays and advanced electron microscopy have however revealed the heterogeneity of recrystallization and the importance of local structural variations for the boundary migration. The migrating recrystallization boundaries are not smooth and do not move in a homogeneous manner with constant speed as described in the classic models, but rather occur with local variations: most segments move in a jerky stop-go manner, with local protrusions and retrusions forming as the boundary migrate [1-3]. Phase-field modeling has shown that the protrusions and retrusions observed on the migrating boundaries may contribute an additional driving force as a result of the boundary curvature and change the migration kinetics on both the local and global scale [4]. Accurate quantitative description of the boundary roughness is necessary in order to evaluate effects hereof on the boundary migration kinetics. Such a description should at least incorporate information about the frequency of protrusions and retrusions as well as their dimensions. One or more parameters that describe this roughness should be developed to allow reasonable comparison among various boundaries.

Mandelbrot has shown that self-similar objects are common in nature and their accurate characterization is of great importance to the field of analytical morphology [5]. Fractals can describe mathematical sets that strictly obey the scaling invariant principle, and fractals are also adopted to describe geometrical objects that exhibit self-similarity. Fractal is generally to be interpreted as a geometrical property describing objects that resembles themselves,



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exactly or statistically, at different observation scales. Since Mandelbrot made the first trial to correlate the fractal dimension of fracture surface roughness with the impact energy absorbed in fracture specimens [6], fractal analysis has been applied in many fields regarding characterizing irregular and rough morphological features [7]. Numerous methods have been proposed and validated in obtaining the fractal dimension [7,8]. These methods basically follow the same algorithm: (1) Measure the quantities of the objects using various step sizes; (2) Plot the measured quantities versus step sizes on logarithm scale and fit a least-squares regression line through the data points; (3) Estimate the fractal dimension as the slope of the regression line. It should be noted that the calculated fractal dimension cannot be expected to be one single value for features that are not strictly self-similar as the Mandelbrot set. Also, the finite resolution of the image data gives a limitation to how small the step sizes can be, so self-similarities can only be observed within a range of scales. However, by investigating the variation of measuring quantities with various step sizes, which is termed fractal analysis in the present work, information is provided of the geometrical features in complex morphologies.

2. Experimental and Fractal Analysis Method

Two cube oriented recrystallizing grains (Figure 1 (a) and (b)) in partly recrystallized microstructures are investigated. Both are from Oxygen Free High Conductivity (OFHC) copper (99.9% purity) 90% cold rolled and annealed at 150 °C. The interior twin boundaries are ignored in the analysis and the boundaries between the recrystallized and the deformed regions are extracted (Figure 1 c and d). The boundary shown in Figure 1(c) is designated as B1 and the boundary shown in Figure 1(d) is designated as B2. Both B1 and B2 have protrusions/retrusions on certain scales, but B2 appears smoother as B1 has more long, narrow branch-like features. Koch snowflake curves [9] of various iterations are included here as a reference for fractal analysis of the two boundaries. The Koch snowflake curve is constructed by iteration, and the geometrical structures are constructed following the same scaling law, whereby it obtains a fractal dimension of around 1.26. Snowflakes with 2 and 3 iterations are shown in Figure 2 and the characteristic edge length of the finest repeating motif is indicated in Figure 2 (c). This length is 675, 225, 75 and 25 pixels for Koch snowflake curves constructed with 2, 3, 4 and 5 iterations, respectively.

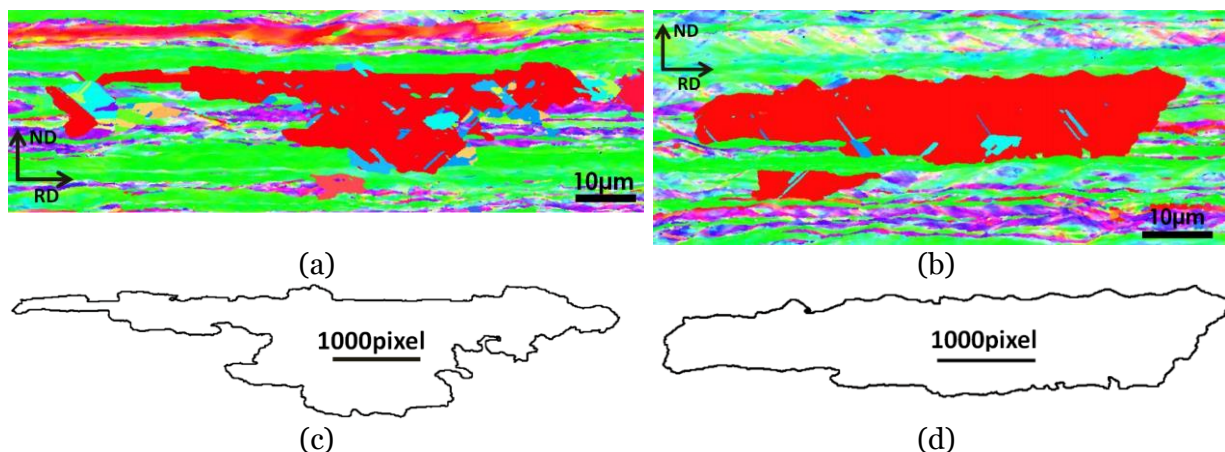


Figure 1. Electron Backscattered Diffraction (EBSD) images measured with step size 0.1 µm of two copper grains. The boundaries surrounding the two grains are shown in (c) and (d), with a pixel bar indicating the scale of digital images; interior twin boundaries are ignored.

The “Minkowski sausage” fractal analysis method [8] follows the algorithm mentioned in the introduction and is used to estimate the fractal behavior in current study. The boundary is dilated with varying widths, and the perimeter is obtained by dividing the area after dilation by the corresponding width. The fractal dimension (FD) is calculated as: $\log(perimeter) = (1 - FD) \log(width) + C$, where FD is the fractal dimension and C is a constant. To obtain more information from the data, as a new idea, the slopes from each set of 2 adjacent points

are also calculated referred to as '2-point fractal dimension (FD)' in the following text. Further information about the method will be published later.

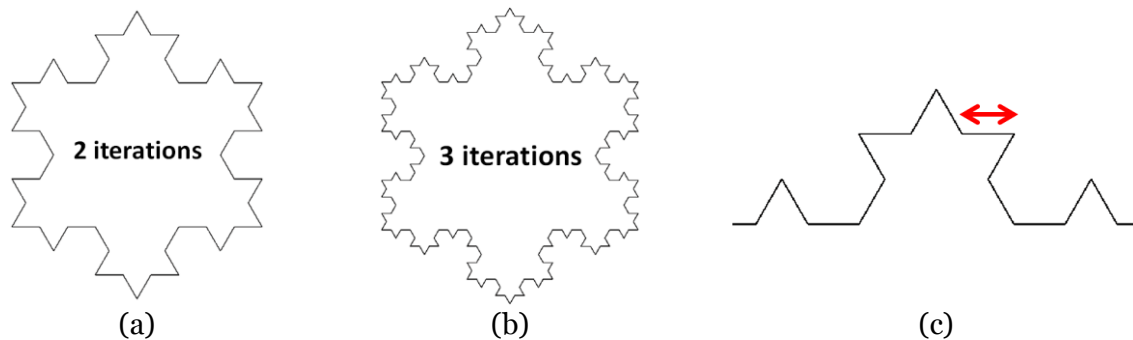


Figure 2. Koch snowflake curve with (a) 2 and (b) 3 iterations. (c) The red arrow indicates the characteristic edge length of the finest motif.

3. Results and Discussion

Results for the Koch snowflake curves with 2, 3, 4 and 5 iterations are shown in Figure 3. Two main points should be noted: (1) For the ideal Koch snowflake curve, the expected $\log(\text{perimeter})$ versus $\log(\text{width})$ plot is a straight line with slope of about -0.26 corresponding to a fractal dimension of 1.26. But with limited iterations, the snowflakes are not ideal fractal geometries and the whole shape is constructed with non-continuous motifs. This result in the inflections of slopes at low $\log(\text{width})$ values in Figure 3(a) and a saw-tooth shape in Figure 3(b); (2) The transitive points as indicated by the arrows ①-④ in Figure 3(b) reveal the positions where the finest motif is resolved by the measurement with increasing width. When a smaller motif is constructed (i.e. more iteration), the transitive point moves towards the smaller width that corresponds with characteristic edge length of the finest motif. This correspondence provides a correlation between the dimension of the features, e.g. finest edge length and the transitive points in the plots.

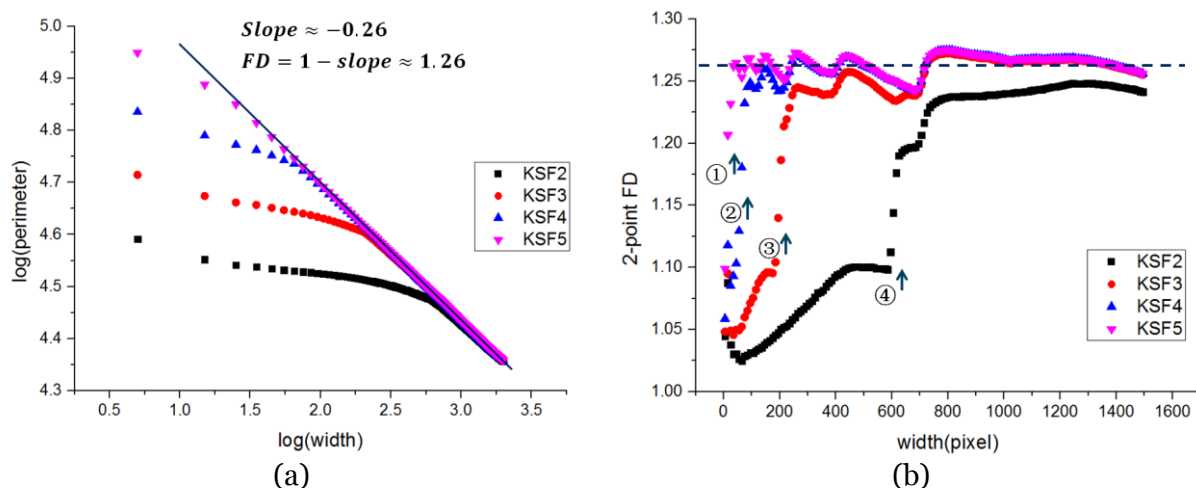


Figure 3 (a) $\log(\text{perimeter})$ versus $\log(\text{width})$ plots and (b) 2-point fractal dimension plot for Koch snowflake curves with 2, 3, 4 and 5 iterations. Arrows ①-④ in (b) mark the transitive points which corresponds to the finest motif of each Koch snowflake curves.

The calculated results for B1 and B2 are shown in Figure 4. It is found that the slope varies in a complicated manner for both B1 and B2. It is not possible to fit a single straight line through the data points and a single fractal dimension value corresponding to the entire boundaries is therefore not found (which agrees with expectation). The arrows in Figure 4(b) mark major transitive points of B1 and B2 curves. For B1, there is an almost steady increase of the 2-point FD values between arrow ⑤ and ⑥. Unlike the sharp increase or saw-tooth behavior in Koch snowflake curves, the increase seen in B1 indicates that there are

protrusions/retrusions distributed over the entire length scale between ⑤ and ⑥, which agrees with the visual impression of the morphology of B1 (see Figure 2(c)). The transitive point ⑥, at the width of about 300 pixels, corresponds to the approximate width of the narrow branches in B1. For B2, two distinct plateaus (between ⑤ and ⑧, ⑨ and ⑩) are seen in Figure 4(b) and they indicate that the protrusions/retrusions are mostly on two scales. Referring to the morphology of B2 (see Figure 2(d)), relative small protrusions/retrusions are mostly on the lower boundary (bottom right) and their dimensions are about 50 pixels corresponding to transitive points marked by ⑤, while the larger protrusions/retrusions are more frequently observed on the upper boundary and their dimensions are about 250 pixels corresponding to ⑨. The 2-point FD value of B1 and B2 shows an almost linear drop at positions ⑦ (width at about 660 pixels) and ⑪ (width at about 500 pixels) respectively, and both widths correspond to the approximate overall geometrical width along the normal direction of B1 and B2, respectively.

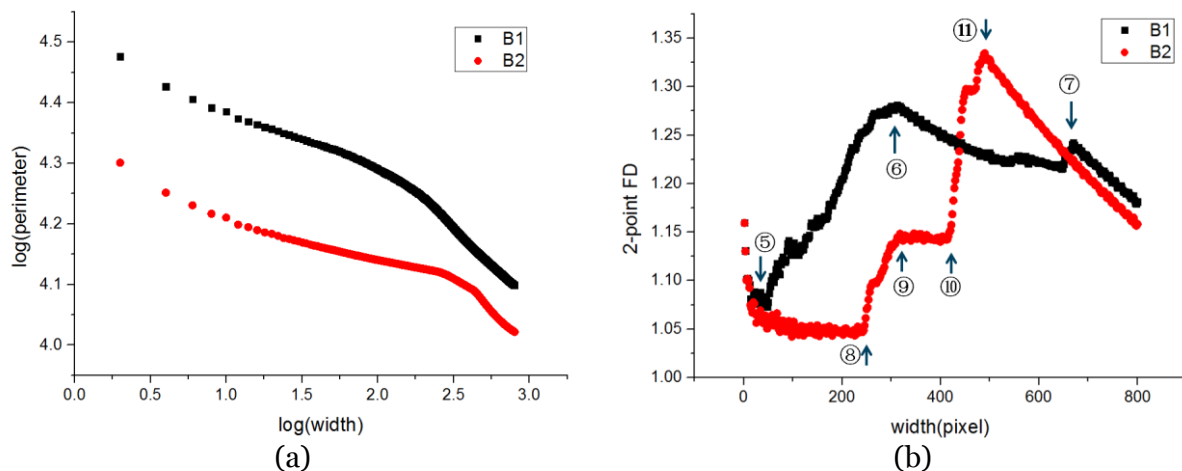


Figure 4. (a) $\log(\text{perimeter})$ versus $\log(\text{width})$ plot and (b) 2-point fractal dimension plot for B1 and B2. Arrows ⑤-⑪ mark the major transitive points for curves of B1 and B2.

4. Conclusions and Outlook

The fractal analysis using the “Minkowski sausage” method provides a way to reveal the characteristics of geometrical shapes, with measurement results reflecting the morphological roughness. The method is adequate to analyze grain boundary roughness but more accurate quantitative description is required and will be discussed in future work.

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